aid of the Legendre-Clebsch necessary condition. This condition may be written (for a minimum)

$$\mu_v \cos\alpha + \mu_\gamma(\sin\alpha/v) \geq 0 \tag{2}$$

In solving the variational boundary value problem, it must be ascertained that inequality (2) is satisfied at each point of the solution.

An intermediate control (i.e., not satisfying the desired boundary conditions) is shown in Fig. 1. As the iterations proceed, the control is reshaped until at some point (A in Fig. 1), α† assumes the value of 90°. At this point $\mu_v = 0$ and $\mu_\gamma > 0$ so that the Legendre-Clebsch condition reduces to

$$C \sin\alpha \geq 0 \qquad C > 0 \tag{3}$$

It is clear that a discontinuity in α from +90° to −90° is not permitted since $\alpha = -90°$ violates condition (3). The arc beyond such a discontinuity is a nonminimal one. A continuous control, on the other hand, does satisfy condition (3). It may be noted that a discontinuity can be allowed only if the equality in Eq. (2) holds. The control given in Fig. 1 everywhere satisfies the Legendre-Clebsch condition.

In closing, it may be emphasized that the origin of the boundary value problem cannot be forgotten in seeking its solution. The Euler-Lagrange equations are not the only conditions that a minimizing arc must satisfy.

References

[1] Greenley, R. R., "Comments on 'The adjoint method and its applications to trajectory optimization,'" AIAA J. 1, 1463 (1963).

[2] Jurovics, S. A. and McIntyre, J. E., "The adjoint method and its application to trajectory optimization," ARS J. 32, 1354–1358 (1962).

[3] Jazwinski, A. H., "Integration of two-point boundary value problems with one unknown boundary," General Dynamics/Astronautics A-PD-113 (May 17, 1962).

[4] Jazwinski, A. H., "New approach to variational problems in trajectory optimization," Martin Co., Space Systems Div., ER 12700 (November 1962).

[5] Jazwinski, A. H., "Optimum trajectories by application of Green's theorem to the variational boundary value problem," Martin Co., Space Systems Div., ER 12943 (March 1963).

† Only a portion of the α-optimal control is shown in Fig. 1.

Comment on "Wind-Tunnel Interference for Wing-Body Combination"

D. R. Holder*
De Havilland Aircraft Company Ltd., Hatfield, Hertfordshire, England

I HAVE studied with interest Gorgui's analysis of the wind-tunnel interference for a wing-body combination.[1] Although I have obtained the same result for a circular tunnel, using the method of images, I believe that the results are misleading and the conclusion inaccurate.

It is usual to allow for the mean interference by means of a correction to the angle of attack, and one is then interested in the spanwise variation of interference which has not been accounted for by this correction.

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* Wind Tunnel Engineer; now Technical Assistant, de Havilland Division of Hawker Siddeley Aviation Ltd.

Now, the upwash induced by a change in angle of attack is not uniform over the span of a wing-body combination. Indeed, by considering uniform flow past a circular cylinder, it can be shown that the upwash is doubled in the vicinity of the wing-body junction. No allowance has been made in Gorgui's analysis for the change in body angle of attack, and this explains the variation in δ near the wing-body junction.

It appears that the curves for $(r/s = 0)$ are valid for determining the correction to angle of attack and the residual interference for a wing-body combination. There is not, as suggested, any tendency towards a root stall, apart from that experienced at the corrected incidence in free flight. This conclusion is important, in view of the importance attached to stall development work in wind tunnels.

Reference

[1] Gorgui, M. A., "Wind-tunnel interference for wing-body combination," J. Aerospace Sci. 28, 823–825 (1961).

Correlation of the Critical Pressure of Conical Shells with That of Equivalent Cylindrical Shells

Josef Singer*
Technion-Israel Institute of Technology, Haifa, Israel

IN Ref. 1, Seide showed that the critical pressures for isotropic conical shells under hydrostatic pressure can be correlated to those of equivalent cylindrical shells. The correlation yielded an approximate curve for the ratio of the critical pressure of conical shells to that of their equivalent cylindrical shells (Fig. 2 of Ref. 1). A very similar curve was obtained in Ref. 2 for conventional simple supports (which differ slightly from Seide's boundary conditions).

However, in both papers the calculations did not include large cone angles. Recent computations indicate that for larger cone angles the single curve should be replaced by a family of curves. Reappraisal of Fig. 2 of Ref. 1 brings out this cone angle dependence for 60°, as can be seen in Fig. 1, where that figure is reproduced with emphasis on the 60° points. (The remainder were for 10°, 20°, 30°, and 45°.) It is apparent that a better fitting correlation curve can be obtained if only cone angles up to 45° are included (or even better if only up to 30°) and the 60° points are joined by a similar curve.

Further computations for conventional simple supports by the method of Ref. 2 yields a family of correlation curves given in Fig. 2. The curves show the ratio of the critical pressure p of a conical shell to that of an equivalent cylindrical shell $\bar{p}$ vs the taper ratio $(1-R_1/R_2)$. The equivalent cylindrical shell is defined as one having the same thickness as the conical shell, but whose radius is the mean radius of curvature of the cone and whose length is that of its slant length. As may be seen, the 60° curve deviates only slightly, whereas the 75° and 85° curves are noticeably lower. The actual percentage reduction in the $(p/\bar{p})$ ratio is only of the order of a few percent (up to about 6–7% for a large taper ratio and a cone angle of 85°), but since it is unconservative it is significant.

The computations brought out another nonconservative secondary effect. Seide's correlation curve[1] and that of Ref.

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* Associate Professor, Department of Aeronautical Engineering. Member AIAA.

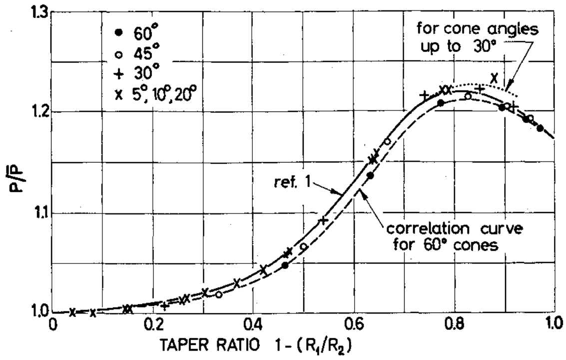

Fig. 1 Correlation between critical pressure of conical and equivalent cylindrical shells with re-emphasis on cone angle dependence (from Fig. 2 of Ref. 1).

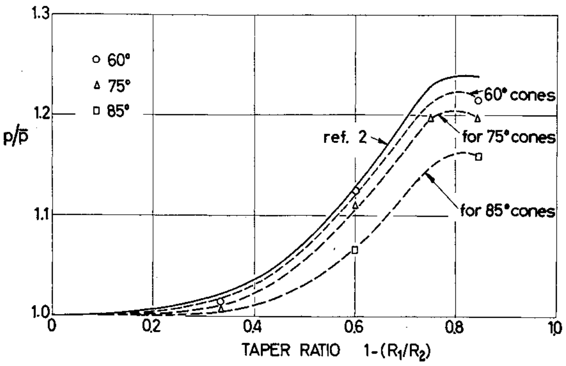

Fig. 2 Correlation curves for conical shells with large cone angles.

2 were based on typical shells of fairly high ratios of small radius to thickness (R_1/h), the range being 250–2000. If one now computes the $(p/\bar{p})$ ratio for a typical shell and then repeats the calculations for the same shell but with different thicknesses, a decrease in $(p/\bar{p})$ with increasing thickness is noted. This decrease is very small for large (a/h) ratios, where (a/h) is an alternative thickness ratio criterion, a being the distance along a generator of the small end of the conical frustrum from the vertex; but it becomes appreciable for thicker shells of (a/h) ratios below 300. For example, although for a typical shell a decrease of only 1.5% in $(p/\bar{p})$ was found when the (a/h) ratio was changed from 700 to 300, a 6% decrease resulted when the (a/h) ratio was changed from 300 to 50.

Since the calculations of Ref. 1 did not go below $(a/h) = 290$, it is not surprising that the decrease of $(p/\bar{p})$ with (a/h), or rather with (R_1/h), was found to be very small and was hence obliterated by averaging out. But, if one intends to apply the correlation curves of Ref. 1 or 2 to shells with (a/h) below 250, this effect may be significant.

It may be pointed out that both cone angle dependence and thickness ratio effects of the same order were found when a similar correlation with equivalent cylindrical shells was carried out for orthotropic shells.[3]

References

[1] Seide, P., "On the buckling of truncated conical shells under uniform hydrostatic pressure," *Proceedings of the IUTAM Symposium on the Theory of Thin Elastic Shells, Delft, Holland, August 1959* (North-Holland Publishing Company, Amsterdam, 1960), p. 363.

[2] Singer, J., "Buckling of conical shells under axisymmetrical external pressure," J. Mech. Eng. Sci. **3**, 330–339 (1961).

[3] Singer, J. and Fersht-Scher, R., "Buckling of orthotropic conical shells under external pressure," Technion Res. and Dev. Foundation, Haifa, Israel, TAE Rept. 22 (September 1962).

Addendum: Dual Electric-Nuclear Engine

Myron Levoy*

Thiokol Chemical Corporation, Denville, N. J.

In a recent paper,[1] comparison was made between all-nuclear rockets and dual electric-nuclear rockets for Mars trips. It was found that the dual electric-nuclear system reduced the gross weight of the vehicle in initial earth orbit to 0.4–0.6 of the gross weight of the all-nuclear vehicle. Comparison was not made, however, with an all-electric vehicle.

A recent parametric study by Moeckel[2] indicates that for a fast round trip comparable to that in Ref. 1. (347 days), and for a power plant specific weight equal to that for the present typical vehicle (8.3 lb/kw), all-electric engines will have about the same gross weight as an all-nuclear vehicle for equal payloads returned to earth. Although details of the mission profile in Moeckel's study are somewhat different, it is believed that the over-all comparisons in the two studies are consistent.

It is therefore possible to conclude that the dual electric-nuclear rocket system would reduce the gross weight of a comparable all-electric vehicle to approximately the same degree (~0.4–0.6) that it would reduce the gross weight of an all-nuclear vehicle.

References

[1] Levoy, M., "Dual electric-nuclear engine," AIAA J. 1, 1298–1302 (1963).

[2] Moeckel, W. E. "Electric propulsion systems for Mars missions," Am. Astronaut. Soc. Symposium on the Exploration of Mars, Preprint No. 4 (June 1963).

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* Head, Non-Chemical Propulsion Section, Preliminary Design, Reaction Motors Division.

Comment on "Velocity Defect Law for a Transpired Turbulent Boundary Layer"

D. S. Hacker*

Institute of Gas Technology, Chicago, Ill.

The interesting note by Mickley and Smith[1] suggests that the hypothesis advanced by Clauser[2] for the turbulent boundary layer can be extended to the problem of the transpired boundary layer by a substitution of the friction velocity $U_\tau{}^*$ based on the maximum shear stress. It has been established that for the simple case of zero axial pressure gradient and for an impermeable plate, $U_\tau{}^*$ is a maximum at the wall. Where a disturbance exists at the wall due to a pressure gradient or indeed mass transport, a single solution of the velocity distribution function is no longer applicable. However, in the outer region of the boundary layer, the momentum equation for the flow is reduced to the Reynolds stress equations:

$$\bar{u}\frac{\partial \bar{u}}{\partial y} + \bar{v}\frac{\partial \bar{u}}{\partial x} = \epsilon\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right) \qquad \text{etc.} \qquad (1)$$

This, in essence, suggests that the eddy diffusivity ϵ of the

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* Supervisor, Combustion Research. Member AIAA.